

# Prerequisites in Mathematics

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The prerequisites for the module **Mathematics** in the Bachelor programme **Tourism** at the University of Applied Sciences of the Grisons are the following:

## Terms

- know the basic arithmetic operations, and be able to perform them.
- be able to rearrange and simplify terms.
- be able to expand and factorise terms.
- know the quadratic binomial theorems, and be able to apply them.
- be able to reduce and expand fractions.
- be able to rearrange and simplify terms with fractions.
- know the power rules, and be able to apply them to powers with integer exponents.
- be able to determine the domain of a term.

## Equations

- be able to solve a linear equation in one variable.

## Functions

- be able to calculate values of a basic function if the equation of the function is given.
- know and understand the graph as a representation of a function.
- be able to determine values and special points if the graph of a basic function is given.
- know and understand what a linear function is.
- know and understand the graph of a linear function.
- be able to treat basic tasks regarding the graph of a linear function.

Concretely, it is expected to be able to solve the problems below **without auxiliaries** (calculator, formulary, etc.):

## Problems

### Terms

1. Calculate the expressions below:

a)  $2 + 3 \cdot 4$

b)  $3^{-2}$

c)  $-2^4$

d)  $\sqrt{16}$

2. Simplify the expressions below:

a)  $7x - 5z + 10y + 3y + 8z - 4x$

b)  $(32m + 13q) - (14m + 7q)$

c)  $(15a - 2b) - [7a - (2a + b)]$

d)  $5a^2b \cdot 4ab \cdot 3a^2b$

e)  $(x^3 - x^2y + xy^2 - y^3)(x + y)$

3. Expand the expressions below:

a)  $(p + q)^2$

b)  $(2x + 3y)^2$

c)  $(x - y)^2$

d)  $(2a - 3ax)^2$

e)  $(a + 2)(a - 2)$

f)  $(5xy + 3xz)(5xy - 3xz)$

4. Factorise the expressions below:

a)  $5a^2 - 10a^3 - 25a^4$

b)  $3a(x - a)^2 + 12a^2(x - a)$

5. Simplify the fractions below by reducing:

a)  $\frac{14a}{18ab}$

b)  $\frac{ab}{a^2b^2c}$

c)  $\frac{8ab}{4a^2 - 4ab}$

d)  $\frac{p^2 + p}{p^2 - 1}$

e)  $\frac{x - y}{y - x}$

6. Rearrange the fractions below such that the denominator is  $10a^2b^2x$ :

a)  $\frac{4y}{2a^2x}$

b)  $\frac{5}{2ax}$

7. Rewrite the expressions below in one fraction:

a)  $\frac{9x}{5} - \frac{6x}{5}$

b)  $\frac{7x - 3y}{a} - \frac{2x + 5y}{a}$

c)  $\frac{x}{2} + \frac{x}{3}$

d)  $\frac{a}{b} - \frac{c}{ab}$

e)  $\frac{a}{a - b} - \frac{b}{a^2 - b^2}$

f)  $\frac{t + 7}{3t - 6} - \frac{t + 4}{t^2 - 2t}$

8. Simplify the expressions below:

a)  $6 \cdot \frac{5}{12}$

b)  $\frac{3}{4a} \cdot \frac{2}{9b}$

c)  $\frac{d - 1}{18d} \cdot \frac{12d^2}{1 - d}$

d)  $\frac{12pqr}{2pr}$

e)  $\frac{16ab + 12aq}{4a}$

f)  $\frac{30a^4b^3c^2}{5a^2bc}$

g)  $\frac{-2x^2 - 4x}{-2x}$

h)  $\frac{ax}{c}$

i)  $\frac{\frac{a}{b^2}}{\frac{a^2}{b}}$

j)  $\frac{x}{\frac{1}{y}}$

k)  $\frac{r^2 + \frac{1}{r}}{r + \frac{1}{r^2}}$

9. Simplify the expressions below and write the answers without fractions:

a)  $\frac{(a^2 b^3 a^4)^5}{(b^2 a^3 b^5)^2}$

b)  $\left(\frac{a^{-1} b^2}{a^{-3} b^4}\right)^{-5}$

10. Determine all real numbers such that the expressions below are **not** defined:

a)  $x^2 - 7$

b)  $\frac{1}{x+2}$

c)  $\sqrt{x+3}$

d)  $\frac{1}{\sqrt{x^2-4}}$

### Equations

11. Solve the equations below for x (without discussing special cases):

a)  $22(x - 11) - 5(x - 40) = 110 - (x + 53)$

b)  $2a + cx = c - x$

c)  $\frac{45}{2x-9} - 2 = -\frac{27}{9-2x}$

d)  $\frac{x}{x-1} - \frac{x-1}{x-2} = 0$

### Functions

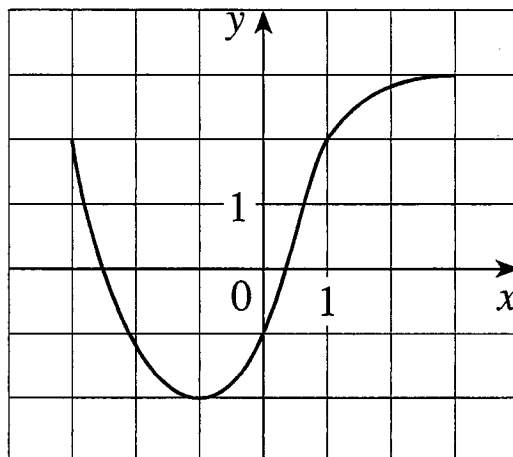
12. The equation of a function f is given as follows:

$$y = f(x) = 3x - 4$$

a) Determine both f(0) and f(-4).

b) Determine all values of x such that f(x) = 0.

13. The graph of a function f is given as follows:



a) Determine f(-1).

b) Estimate the value of f(2).

- c) Determine the values of  $x$  such that  $f(x) = 2$ .  
 d) Estimate the values of  $x$  such that  $f(x) = 0$ .

14. The graph of a linear function contains the two points  $P(-2|5)$  and  $Q(2|-4)$ .  
 a) Determine the equation of the function.  
 b) Determine the point where the graph and the  $y$ -axis intersect.  
 c) Determine the point where the graph and the  $x$ -axis intersect.

## Answers

### Terms

- |    |    |                                  |    |                            |
|----|----|----------------------------------|----|----------------------------|
| 1. | a) | 14                               | b) | $\frac{1}{9}$              |
|    | c) | -16                              | d) | 4                          |
| 2. | a) | $3x + 13y + 3z$                  | b) | $18m + 6q$                 |
|    | c) | $10a - b$                        | d) | $60a^5b^3$                 |
|    | e) | $x^4 - y^4$                      |    |                            |
| 3. | a) | $p^2 + 2pq + q^2$                | b) | $4x^2 + 12xy + 9y^2$       |
|    | c) | $x^2 - 2xy + y^2$                | d) | $4a^2 - 12a^2x + 9a^2x^2$  |
|    | e) | $a^2 - 4$                        | f) | $25x^2y^2 - 9x^2z^2$       |
| 4. | a) | $5a^2(1 - 2a - 5a^2)$            | b) | $3a(x - a)(x + 3a)$        |
| 5. | a) | $\frac{7}{9b}$                   | b) | $\frac{1}{abc}$            |
|    | c) | $\frac{2b}{a - b}$               | d) | $\frac{p}{p - 1}$          |
|    | e) | -1                               |    |                            |
| 6. | a) | $\frac{20b^2y}{10a^2b^2x}$       | b) | $\frac{25ab^2}{10a^2b^2x}$ |
| 7. | a) | $\frac{3x}{5}$                   | b) | $\frac{5x - 8y}{a}$        |
|    | c) | $\frac{5x}{6}$                   | d) | $\frac{a^2 - c}{ab}$       |
|    | e) | $\frac{a^2 + ab - b}{a^2 - b^2}$ | f) | $\frac{t + 6}{3t}$         |

8. a)  $\frac{5}{2}$  b)  $\frac{1}{6ab}$   
 c)  $-\frac{2d}{3}$  d)  $6q$   
 e)  $4b + 3q$  f)  $6a^2b^2c$   
 g)  $x + 2$  h)  $\frac{x}{c}$   
 i)  $\frac{1}{ab}$  j)  $xy$   
 k)  $r$
9. a)  $a^{24}b$  b)  $a^{-10}b^{10}$
10. a) The expression is defined for all real numbers  $x$ .  
 b)  $x = -2$   
 c)  $x < -3$   
 d)  $-2 \leq x \leq 2$

### Equations

11. a)  $x = \frac{11}{2}$   
 b)  $x = \frac{c-2a}{1+c}$   
 c)  $x = 9$   
 d) The equation has no solution.

### Functions

12. a)  $f(0) = -4$   
 $f(-4) = -16$   
 b)  $x = \frac{4}{3}$
13. a)  $f(-1) = -2$   
 b)  $f(2) \approx 2.8$   
 c)  $x_1 = -3$   
 $x_2 = 1$   
 d)  $x_1 \approx -2.5$   
 $x_2 \approx 0$

14. a)  $y = f(x) = -\frac{9}{4}x + \frac{1}{2}$   
b)  $S_y\left(0 \mid \frac{1}{2}\right)$   
c)  $S_x\left(\frac{2}{9} \mid 0\right)$